FACULTY OF AVIATION AND AUTOMOTIVE SYSTEMS (LFS)

HAW HAMBURG

# Induced Drag of Tapered Wings – Revisiting Hoerner's Approach with AVL and VSPAERO

## The span efficiency factor, *e* is not constant, but a function of and taper ratio, λ <u>and</u> aspect ratio, *A* .

**Induced drag is calculated from $C_{D,i} = C_L^2/(\pi Ae)$. Clearly, induced drag can be reduced by increasing the aspect ratio, A. For elliptical wings, e = 1. Tapered wings have e < 1, even at best taper ratio, $\lambda = c_t/c_r$ of 0.45 (NASA Report 921) respectively 0.357 (Hoerner 1965). With Hoerner's f(λ) it is possible to calculate e = 1/(1+f(λ) A). More is possible ...**

### PURPOSE

Calculate span efficiency factors (e) of straight (unswept) wings for various aspect ratios (A) and taper ratios (0 < λ < 1) using computational fluid dynamics (CFD). Use the notation from Hoerner (Figure 1). From CFD results calculate e and continue to calculate f(λ) = (1-e)/(Ae). Users of the results can calculate back to the span efficiency factor with e = 1/(1+f(λ) A). See Figure 2.

### METHODOLOGY

Wings are modeled in AVL and VSPAERO (VLM and Panel Method) with different aspect ratios and taper ratios. At a selected angle of attack, the lift coefficient and the induced drag coefficient are calculated. From the CFD results the span efficiency factor, e and f(λ) are calculated. f(λ) is plotted in comparison to Hoerner's (theoretical) curve (Figure 3 and Figure 4).

### FINDINGS

Span efficiency factor, e <u>decreases</u> with increasing aspect ratio, A (Figure 2). The panel method from VSPAERO shows curves approaching Hoerner's curve with increasing aspect ratio (Figure 3). Calculated values of f(λ) are larger resulting in lower span efficiency factor (e). The Vortex Lattice Method (VLM) offered by AVL also shows curves approaching Hoerner's curve with increasing aspect ratio. However, calculated values of f(λ) are smaller (Figure 4, right) resulting in a larger span efficiency factor (e). The VLM of VSPAERO shows curves f(λ) of similar magnitude as Hoerner, but without clear tendencies. Shown is only the result for A = 10 (Figure 4, left).

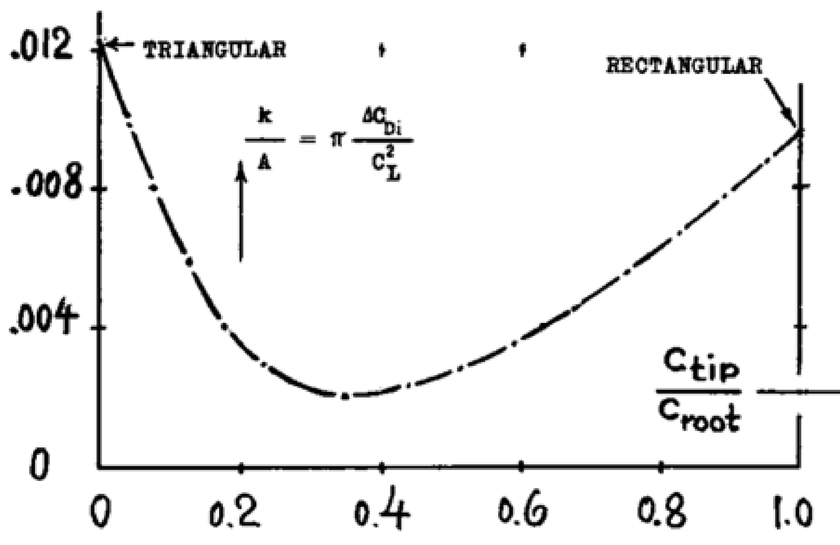
### PRACTICAL IMPLICATIONS

Hoerner's curve seems to be a helpful and simple means to calculate the span efficiency factor (e). To assume that f(λ) is independent of aspect ratio may be a simplification. A hypothesis is that Hoerner's curve represents values for an aspect ratio approaching infinity.

### ORIGINALITY

Results calculated with AVL were published by Budziak (2015) and with VSPAERO by Mariën (2021), both supervised by Scholz. This is the first presentation at a conference.

$$f(\lambda) = 0.0524\,\lambda^4 - 0.15\lambda^3 + 0.1659\lambda^2 - 0.0706\lambda + 0.0119$$



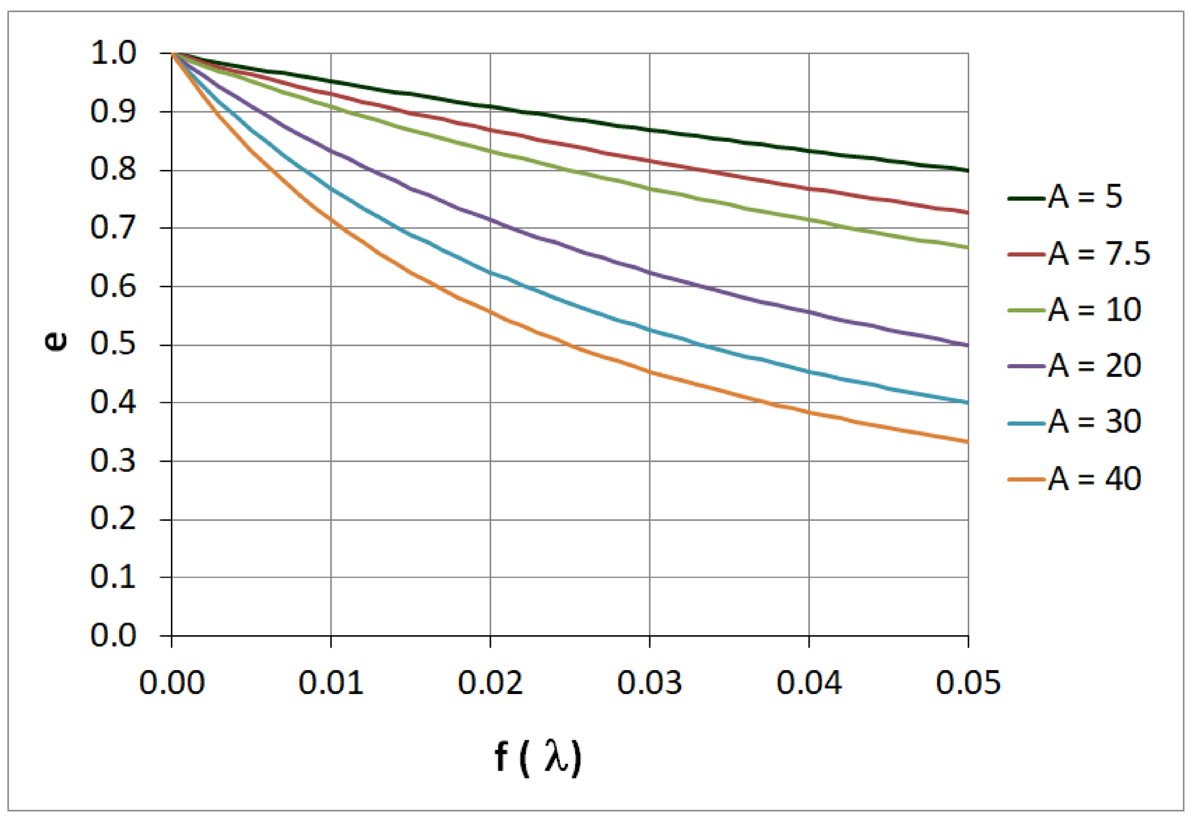

$$C_{D,i} = \frac{(1+k)C_L^2}{\pi A} \qquad f(\lambda) = \frac{k}{A}$$

$$C_{D,i} = \frac{C_L^2}{\pi A}\left[1 + Af(\lambda)\right]$$

$$C_{D,i} = \frac{C_L^2}{\pi}\left[\frac{1}{A} + f(\lambda)\right] \qquad \Delta C_{D,i} = \frac{C_L^2}{\pi} f(\lambda)$$

$$e = \frac{1}{1 + Af(\lambda)} \qquad e = \frac{1}{1+k}$$

**Figure 1:** Hoerner's (theoretical) curve, f(λ) is independent of aspect ratio, A. The hypothesis is that the curve represents an aspect ratio, A towards infinity.



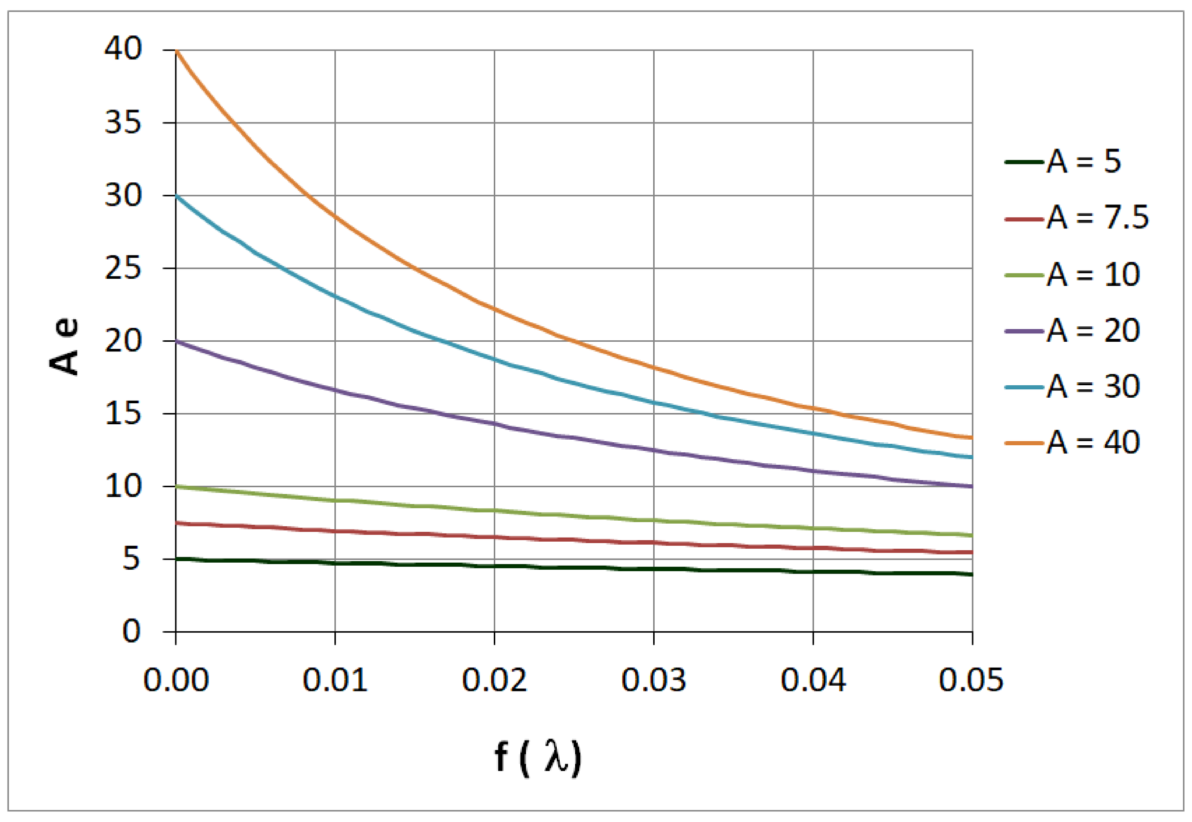
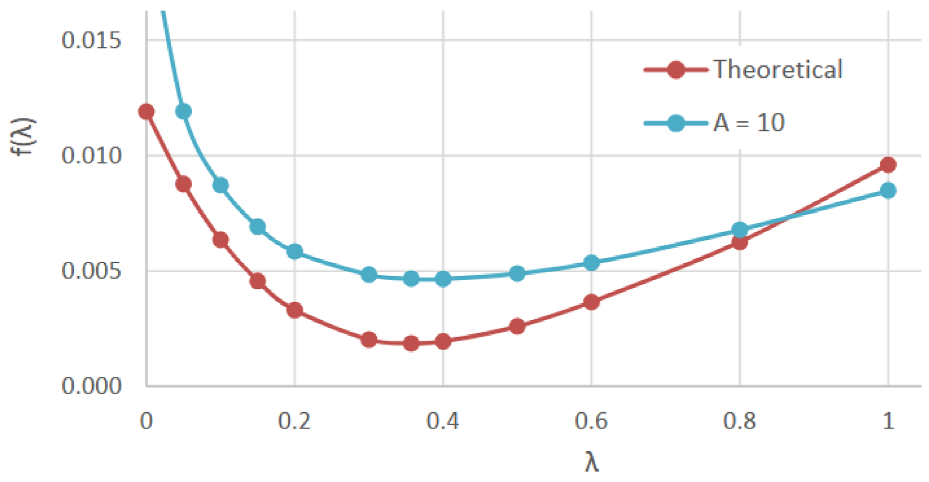




**Figure 2:** Span efficiency factor calculated from e = 1/(1+f(λ) A). e decreases with A. In contrast A e increases with A. For f(λ) = 0 the span efficiency factor, e = 1. A e and e go down with f(λ) .

$$C_{D,i} = \frac{C_L^2}{\pi Ae} \qquad e = \frac{1}{1 + Af(\lambda)}$$



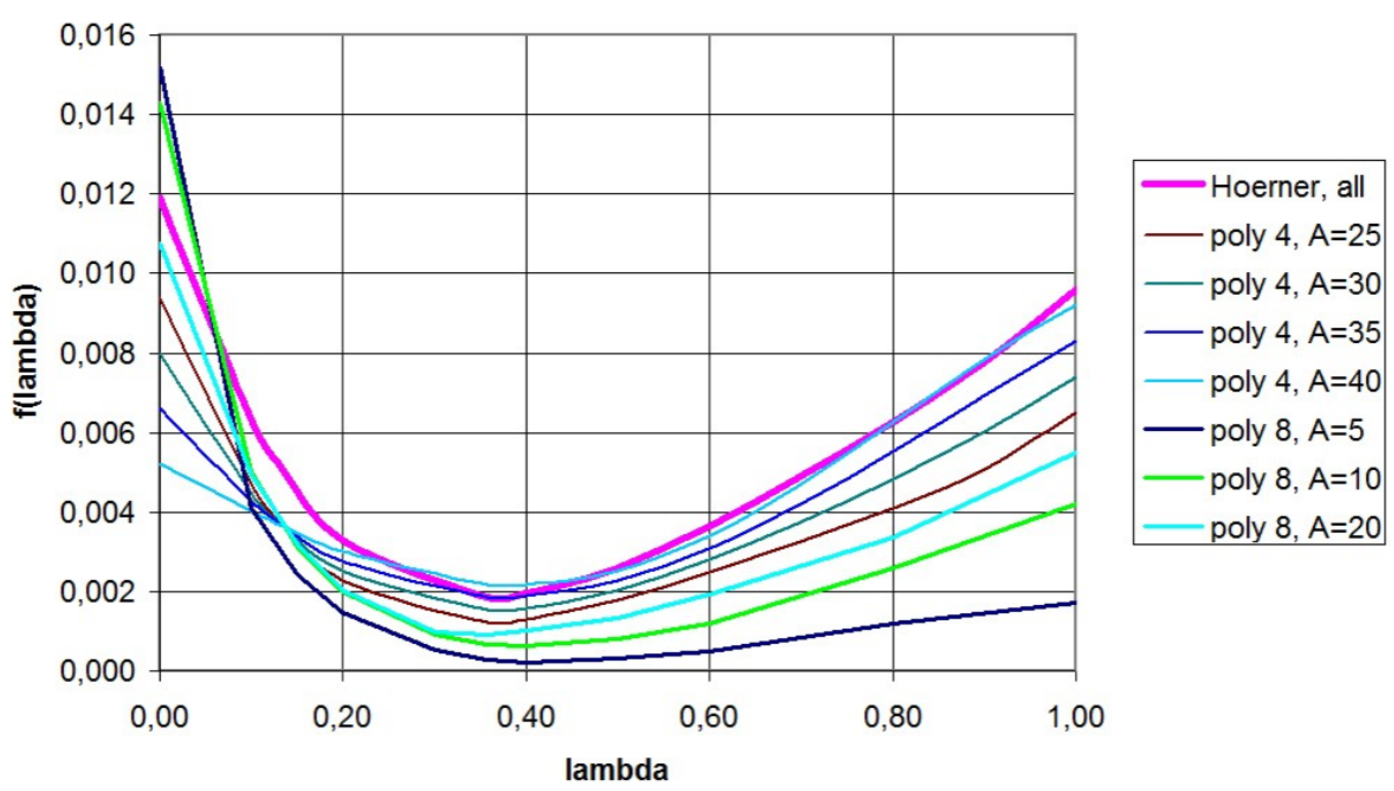




**Figure 4:** VLM: *Left*: Result from VSPAERO is <u>above</u> "Hoerner / theoretical"- *Right*: Results from AVL for all aspect ratios, A are <u>below</u> Hoerner.

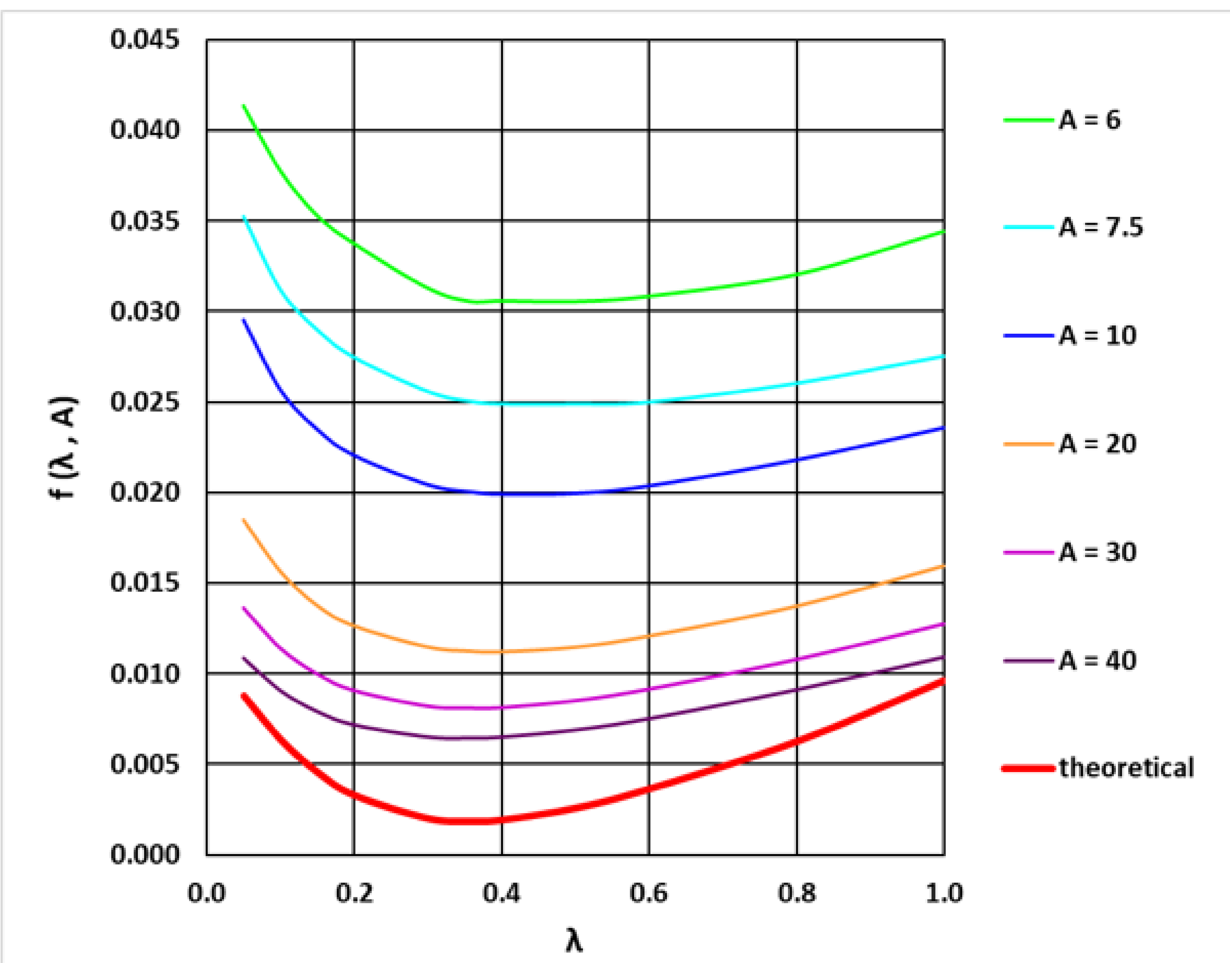



**Figure 3:** Panel Method. Results from VSPAERO for all aspect ratios, A are <u>above</u> "Hoerner / theoretical".

**More details in the Project of Budziak (2015):**
https://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2015-09-20.015

**More details in the Master Thesis of Mariën (2019):**
https://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2021-07-16.018

**Associated research data (Harvard Dataverse):**
https://doi.org/10.7910/DVN/0S1R14


**Kinga Budziak**
**Floris Mariën**

**Prof. Dr. Dieter Scholz, MSME**

HOCHSCHULE FÜR ANGEWANDTE WISSENSCHAFTEN HAMBURG
Hamburg University of Applied Sciences



DGLR
DLRK 2026
DEUTSCHER LUFT- UND RAUMFAHRTKONGRESS
08. – 10.09.2026 | AACHEN